

3.4

Ασκήσεις σχολικού βιβλίου σελίδας 333

A' Ομάδας

1.

Av $\int_1^4 f(x)dx = 9$, $\int_3^4 f(x)dx = 11$, $\int_1^8 f(x)dx = 13$, va βρείτε τα ολοκληρώματα

i) $\int_4^3 f(x)dx$

ii) $\int_4^8 f(x)dx$

iii) $\int_1^3 f(x)dx$

iv) $\int_3^8 f(x)dx$

Λύση

i)

$$\int_4^3 f(x)dx = - \int_3^4 f(x)dx = -11$$

ii)

$$\int_4^8 f(x)dx = \int_4^1 f(x)dx + \int_1^8 f(x)dx = -\int_1^4 f(x)dx + \int_1^8 f(x)dx = -9 + 13 = 4$$

iii)

$$\int_1^3 f(x)dx = \int_1^4 f(x)dx + \int_4^3 f(x)dx = 9 - 11 = -2$$

iv)

$$\int_3^8 f(x)dx = \int_3^4 f(x)dx + \int_4^8 f(x)dx = 11 + 4 = 15$$

2.

Να αποδείξετε ότι $\int_1^e \ln t dt = \int_e^1 \ln \frac{1}{t} dt$

Λύση

$$\int_e^1 \ln \frac{1}{t} dt = \int_e^1 \ln t^{-1} dt = \int_e^1 -\ln t dt = -\int_e^1 \ln t dt = \int_1^e \ln t dt$$

3.

$$\text{Να υπολογίσετε το κέτσι ώστε} \quad \int_1^{\kappa} \frac{x^2 - 4}{x^2 + 1} dx - \int_{\kappa}^1 \frac{5}{x^2 + 1} dx = 3$$

Λύση

$$\int_1^{\kappa} \frac{x^2 - 4}{x^2 + 1} dx - \int_{\kappa}^1 \frac{5}{x^2 + 1} dx = 3 \quad \Leftrightarrow \quad \int_1^{\kappa} \frac{x^2 - 4}{x^2 + 1} dx + \int_1^{\kappa} \frac{5}{x^2 + 1} dx = 3$$

$$\int_1^{\kappa} \left(\frac{x^2 - 4}{x^2 + 1} + \frac{5}{x^2 + 1} \right) dx = 3$$

$$\int_1^{\kappa} \frac{x^2 - 4 + 5}{x^2 + 1} dx = 3$$

$$\int_1^{\kappa} \frac{x^2 + 1}{x^2 + 1} dx = 3$$

$$\int_1^{\kappa} 1 dx = 3$$

$$1(\kappa - 1) = 3 \Leftrightarrow \kappa = 4$$

4.

$$\text{Αν} \quad \int_1^3 f(x) dx = 5 \quad \text{και} \quad \int_1^3 g(x) dx = -2 \quad \text{να υπολογίσετε τα ολοκληρώματα}$$

$$\text{i)} \quad \int_1^3 (2f(x) - 6g(x)) dx \quad \text{και} \quad \text{ii)} \quad \int_3^1 (2f(x) - g(x)) dx$$

Λύση**i)**

$$\begin{aligned} \int_1^3 (2f(x) - 6g(x)) dx &= -2 \int_1^3 f(x) dx - 6 \int_1^3 g(x) dx \\ &= 2 \cdot 5 - 6 \cdot (-2) \\ &= 10 + 12 = 22 \end{aligned}$$

ii)

$$\begin{aligned} \int_3^1 (2f(x) - g(x)) dx &= - \int_1^3 (2f(x) - g(x)) dx \\ &= -2 \int_1^3 f(x) dx + \int_1^3 g(x) dx \\ &= -2 \cdot 5 + (-2) \\ &= -10 - 2 = -12 \end{aligned}$$