5.1 ΑΚΟΛΟΥΘΙΕΣ

Ασκήσεις σχολικού βιβλίου σελίδας 124

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1.

Να βρείτε τους πέντε πρώτους όρους των ακολουθιών :

i) $\alpha_\nu = 2\nu + 1$, **ii)** $\alpha_\nu = 2^\nu$, **iii)** $\alpha_\nu = \nu^2 + \nu$, **iv)** $\alpha_\nu = \dfrac{\nu^2 - 1}{\nu + 1}$

v) $\alpha_\nu = \left(-\dfrac{1}{10}\right)^{\nu-1}$ **vi)** $\alpha_\nu = 1 - \left(-\dfrac{1}{2}\right)^\nu$, **vii)** $\alpha_\nu = |5-\nu|$, **viii)** $\alpha_\nu = \eta\mu\dfrac{\nu\pi}{4}$

ix) $\alpha_\nu = \dfrac{2^\nu}{\nu^2}$, **x)** $\alpha_\nu = (-1)^{\nu+1}\cdot\dfrac{1}{\nu}$, **xi)** $\alpha_\nu = (-1)^{\nu+1}$

Λύση

i)

$\alpha_1 = 2\cdot1 + 1 = 3$, $\alpha_2 = 2\cdot2 + 1 = 5$, $\alpha_3 = 2\cdot3 + 1 = 7$,

$\alpha_4 = 2\cdot4 + 1 = 9$, $\alpha_5 = 2\cdot5 + 1 = 11$

ii)

$\alpha_1 = 2^1 = 2$, $\alpha_2 = 2^2 = 4$, $\alpha_3 = 2^3 = 8$, $\alpha_4 = 2^4 = 16$, $\alpha_5 = 2^5 = 32$

iii)

$\alpha_1 = 1^2 + 1 = 2$ και ομοίως $\alpha_2 = 6$, $\alpha_3 = 12$, $\alpha_4 = 20$, $\alpha_5 = 30$

iv)

$\alpha_1 = \dfrac{1^2 - 1}{1+1} = 0$ και ομοίως $\alpha_2 = 1$, $\alpha_3 = 2$, $\alpha_4 = 3$, $\alpha_5 = 4$

v)

$\alpha_1 = \left(-\dfrac{1}{10}\right)^{1-1} = 1$, $\alpha_2 = \left(-\dfrac{1}{10}\right)^{2-1} = -\dfrac{1}{10}$, $\alpha_3 = \dfrac{1}{100}$,

$\alpha_4 = -\dfrac{1}{1000}$, $\alpha_5 = \dfrac{1}{10000}$

vi)

$\alpha_1 = 1 - \left(-\dfrac{1}{2}\right)^1 = 1 + \dfrac{1}{2} = \dfrac{3}{2}$, $\alpha_2 = 1 - \left(-\dfrac{1}{2}\right)^2 = 1 - \dfrac{1}{4} = \dfrac{3}{4}$,

$\alpha_3 = 1 - \left(-\dfrac{1}{2}\right)^3 = 1 + \dfrac{1}{8} = \dfrac{9}{8}$, $\alpha_4 = 1 - \left(-\dfrac{1}{2}\right)^4 = 1 - \dfrac{1}{16} = \dfrac{15}{16}$,

$\alpha_5 = 1 - \left(-\dfrac{1}{2}\right)^5 = 1 + \dfrac{1}{32} = \dfrac{33}{32}$

vii)

$\alpha_1 = |5-1| = 4,$ $\alpha_2 = |5-2| = 3,$ $\alpha_3 = |5-3| = 2,$

$\alpha_4 = |5-4| = 1,$ $\alpha_1 = |5-5| = 0$

viii)

$\alpha_1 = \eta\mu\dfrac{1\cdot\pi}{4} = \dfrac{\sqrt{2}}{2}$, $\alpha_2 = \eta\mu\dfrac{2\cdot\pi}{4} = 1$, $\alpha_3 = \eta\mu\dfrac{3\cdot\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\alpha_4 = \eta\mu\dfrac{4\cdot\pi}{4} = 0$, $\alpha_5 = \eta\mu\dfrac{5\cdot\pi}{4} = -\dfrac{\sqrt{2}}{2}$

ix)

$\alpha_1 = \dfrac{2^1}{1^2} = 2$, $\alpha_2 = \dfrac{2^2}{2^2} = 1$, $\alpha_3 = \dfrac{2^3}{3^2} = \dfrac{8}{9}$,

$\alpha_4 = \dfrac{2^4}{4^2} = \dfrac{16}{16} = 1$, $\alpha_5 = \dfrac{2^5}{5^2} = \dfrac{32}{25}$

x)

$\alpha_1 = (-1)^{1+1}\cdot\dfrac{1}{1} = 1$, $\alpha_2 = (-1)^{2+1}\cdot\dfrac{1}{2} = -\dfrac{1}{2}$, $\alpha_3 = (-1)^{3+1}\cdot\dfrac{1}{3} = \dfrac{1}{3}$

$\alpha_4 = (-1)^{4+1}\cdot\dfrac{1}{4} = -\dfrac{1}{4}$, $\alpha_5 = (-1)^{5+1}\cdot\dfrac{1}{5} = \dfrac{1}{5}$

xi)

$\alpha_1 = (-1)^{1+1} = 1$, $\alpha_2 = (-1)^{2+1} = -1$, $\alpha_3 = (-1)^{3+1} = 1$,

$\alpha_4 = (-1)^{4+1} = -1$, $\alpha_5 = (-1)^{5+1} = 1$

2.

Να βρείτε τους πέντε πρώτους όρους των ακολουθιών

i) $\alpha_1 = 2,\ \ \alpha_{\nu+1} = \dfrac{1}{\alpha_\nu}$ **ii)** $\alpha_1 = 0,\ \ \alpha_{\nu+1} = \alpha_\nu^2 + 1$ **iii)** $\alpha_1 = 3,\ \ \alpha_{\nu+1} = 2(\alpha_\nu - 1)$

Λύση

i)

$\alpha_1 = 2,$ $\alpha_2 = \dfrac{1}{\alpha_1} = \dfrac{1}{2}$, $\alpha_3 = \dfrac{1}{\alpha_2} = \dfrac{1}{\frac{1}{2}} = 2$, $\alpha_4 = \dfrac{1}{\alpha_3} = \dfrac{1}{2}$, $\alpha_5 = \dfrac{1}{\alpha_4} = \dfrac{1}{\frac{1}{2}} = 2$

ii)

$\alpha_1 = 0,$ $\alpha_2 = \alpha_1^2 + 1 = 0 + 1 = 1,$ $\alpha_3 = \alpha_2^2 + 1 = 1 + 1 = 2$

$\alpha_4 = \alpha_3^2 + 1 = 4 + 1 = 5,$ $\alpha_5 = \alpha_4^2 + 1 = 25 + 1 = 26$

iii)

$\alpha_1 = 3,$ $\alpha_2 = 2(\alpha_1 - 1) = 2\cdot 2 = 4$, $\alpha_3 = 2(\alpha_2 - 1) = 2\cdot 3 = 6$

$\alpha_4 = 2(\alpha_3 - 1) = 2\cdot 5 = 10$, $\alpha_5 = 2(\alpha_4 - 1) = 2\cdot 9 = 18$

3.

Να ορίσετε αναδρομικά τις ακολουθίες

i) $\alpha_\nu = \nu + 5$, **ii)** $\alpha_\nu = 2^\nu$, **iii)** $\alpha_\nu = 2^\nu - 1$, **iv)** $\alpha_\nu = 5\nu + 3$

Λύση

i)

$\alpha_1 = 6$ και $\alpha_{\nu+1} - \alpha_\nu = (\nu + 1) + 5 - (\nu + 5)$ $\Rightarrow$ $\alpha_1 = 6$ και $\alpha_{\nu+1} - \alpha_\nu = 1$

$\Rightarrow$ $\alpha_1 = 6$ και $\alpha_{\nu+1} = 1 + \alpha_\nu$ με $\nu \in \mathbb{N}^*$

ii)

$\alpha_1 = 2$ και $\dfrac{\alpha_{\nu+1}}{\alpha_\nu} = \dfrac{2^{\nu+1}}{2^\nu}$ $\Rightarrow$ $\alpha_1 = 2$ και $\dfrac{\alpha_{\nu+1}}{\alpha_\nu} = 2$

$\alpha_1 = 2$ και $\alpha_{\nu+1} = 2\alpha_\nu$ με $\nu \in \mathbb{N}^*$

iii)

$\alpha_1 = 1$ και $\alpha_{\nu+1} - \alpha_\nu = 2^{\nu+1} - 1 - 2^\nu + 1$ $\Rightarrow$ $\alpha_1 = 1$ και $\alpha_{\nu+1} - \alpha_\nu = 2^\nu(2 - 1)$

$\alpha_1 = 1$ και $\alpha_{\nu+1} - \alpha_\nu = 2^\nu$ **(1)**

Από την υπόθεση $\alpha_\nu = 2^\nu - 1$ παίρνουμε $2^\nu = \alpha_\nu + 1$

Η (1) $\Rightarrow$ $\alpha_1 = 1$ και $\alpha_{\nu+1} - \alpha_\nu = \alpha_\nu + 1$ $\Rightarrow$

$\alpha_1 = 1$ και $\alpha_{\nu+1} = 2\alpha_\nu + 1$ με $\nu \in \mathbb{N}^*$

iv)

$\alpha_1 = 8$ και $\alpha_{\nu+1} - \alpha_\nu = 5(\nu + 1) + 3 - (5\nu + 3)$ $\Rightarrow$

$\alpha_1 = 8$ και $\alpha_{\nu+1} - \alpha_\nu = 5\nu + 5 + 3 - 5\nu - 3$ $\Rightarrow$

$\alpha_1 = 8$ και $\alpha_{\nu+1} = \alpha_\nu + 5$ με $\nu \in \mathbb{N}^*$

4.

Να βρείτε το ν-οστό όρο των ακολουθιών

i) $\alpha_1 = 1$ και $\alpha_{\nu+1} = \alpha_\nu + 2$ **ii)** $\alpha_1 = 3$ και $\alpha_{\nu+1} = 5\alpha_\nu$

Λύση

i)

$\alpha_1 = \alpha_1$

$\alpha_2 = \alpha_1 + 2$

$\alpha_3 = \alpha_2 + 2$

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$\alpha_{\nu-1} = \alpha_{\nu-2} + 2$

$\alpha_\nu = \alpha_{\nu-1} + 2$

Με πρόσθεση κατά μέλη $\Rightarrow$ $\alpha_\nu = \alpha_1 + 2(\nu - 1)$

$\alpha_\nu = 1 + 2\nu - 2$

$\alpha_\nu = 2\nu - 1$

ii)

$\alpha_1 = \alpha_1$

$\alpha_2 = 5\alpha_1$

$\alpha_3 = 5\alpha_2$

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$\alpha_{\nu-1} = 5\alpha_{\nu-2}$

$\alpha_\nu = 5\alpha_{\nu-1}$

Με πολλαπλασιασμό κατά μέλη έχουμε $\alpha_\nu = 5^{\nu-1}\alpha_1 \Rightarrow \alpha_\nu = 3 \cdot 5^{\nu-1}$.